Published by Institute of Physics Publishing for SISSA/ISAS
Received: *January 16, 2003*
Accepted: *February 7, 2003*

Value of α_{s} from deep-inelastic-scattering data

S. I. Alekhin
Institute for High Energy Physics
142281 Protvino, Russia
E-mail: alekhin@sirius.ihep.su

Abstract: We report the value of α_{s} obtained from QCD analysis of existing data on deep-inelastic scattering of charged leptons off proton and deuterium and estimate its theoretical uncertainties with particular attention paid to impact of the high-twist contribution to the deep-inelastic-scattering structure functions. Taking into account the major uncertainties the value $\alpha_{\mathrm{s}}^{\mathrm{NNLO}}(M_{\mathrm{Z}}) = 0.1143 \pm 0.0014(\mathrm{exp.}) \pm 0.0013(\mathrm{theor.})$ is obtained. An extrapolation of the LO–NLO–NNLO results to the higher orders makes it possible to estimate $\alpha_{\mathrm{s}}^{\mathrm{N^3LO}}(M_{\mathrm{Z}}) \sim 0.113$.

Keywords: QCD, Deep Inelastic Scattering.

Contents

1. Introduction

The value of strong coupling constant α_{s} can be extracted from the existing data on deep-inelastic-scattering (DIS) with experimental uncertainty at the one-percent level. With such experimental accuracy achieved the theoretical uncertainties become dominant. One of the major theoretical uncertainty in α_{s} derived from the next-to-leading-order (NLO) QCD analysis of DIS data comes from the higher-order (HO) corrections. Therefore, consideration of the next-to-next-to-leading (NNLO) corrections is of great importance for suppression of the errors in α_{s}. Indeed, account of the NNLO terms in the QCD evolution equations for the DIS structure functions, which have been calculated recently [1, 2], allows to reduce the HO uncertainty in α_{s} by a factor of $2 \div 3$, as it was estimated in ref. [3]. Another important source of the uncertainty in α_{s} is the high-twist (HT) contribution to the structure functions. This contribution inevitably appear in the operator product expansion [4], but unfortunately cannot be reliably estimated hitherto. In the limited range of the momentum transfer Q the HT terms, which fall with Q by the power law, can simulate the logarithmic-type behavior of the leading twist (LT) term. For this reason the magnitude of HT terms is correlated with the value of α_{s}, which defines the slope of the LT term, and the uncertainty in HT terms propagates into the uncertainty of α_{s}.

We study impact of these sources of uncertainties on the value of α_{s} obtained from a NNLO QCD analysis of existing data on the charged-leptons DIS off the proton and deuterium targets [5]. The data outside the kinematical region of $Q^2 = 2.5 \div 300~\mathrm{GeV}^2$ and $x = 10^{-4} \div 0.75$ are left out to suppress potentially dangerous theoretical uncertainties keeping sufficient sensitivity of data to the value of α_{s}. The data are analyzed using a numerical integration of the QCD evolution equations in the x-space (details are described elsewhere [6]). The value of α_s obtained in the recent update of this fit is [7]

$$\alpha_{\mathrm{s}}^{\mathrm{NNLO}}(M_{\mathrm{Z}}) = 0.1143 \pm 0.0014(\mathrm{exp.})\,, \tag{1.1}$$

to be compared with

$$\alpha_{\mathrm{s}}^{\mathrm{NNLO}}(M_{\mathrm{Z}}) = 0.1166 \pm 0.0009(\mathrm{exp.}) \tag{1.2}$$

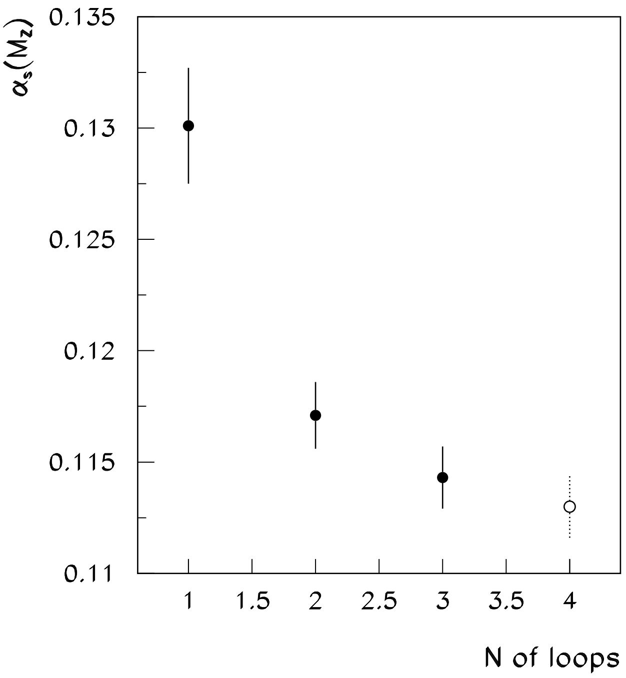

Figure 1: The central values of $\alpha_s(M_Z)$ and the experimental errors obtained in different orders of pQCD (full symbols). The open symbol shows the result of extrapolation to the N^3LO.

obtained by Santiago and Yndurain from the analysis of the proton subset of data [8]. The achieved experimental error in α_s is quite small and therefore, complimentary analysis of theoretical uncertainties is necessary in order to clarify the confident range of α_s variation.

LO	$0.1301 \pm 0.0026(\text{exp.}) \pm 0.0149(\text{RS})$
NLO	$0.1171 \pm 0.0015(\text{exp.}) \pm 0.0033(\text{RS})$
NNLO	$0.1143 \pm 0.0014(\text{exp.}) \pm 0.0009(\text{RS})$

Table 1: The values of $\alpha_s(M_Z)$ obtained in different orders of pQCD analysis and their RS errors.

2. Uncertainty due to the higher-order corrections

Generally, estimate of the HO uncertainty is impossible before the HO corrections are known. However, the approach based on variation of the QCD renormalization scale (RS) is commonly used for estimation of the HO uncertainties. This approach is very approximate since it does not account for a possible x-dependence of the corresponding scale-factor and since the variation range of the scale-factor is optional. Nevertheless, following this approach we estimate the HO uncertainty as the change of α_s caused by variation of the QCD RS from Q to $2Q$. The estimates obtained are given in table 1. Note the reduction of α_s with the pQCD order approximately coincide with the corresponding HO uncertainties[1]. Extrapolating this regularity to the N^3LO we have an estimate $\alpha_s^{N^3LO}(M_Z) \sim 0.113$.

[1]The decrease in α_s because of going from the NLO to the NNLO disagrees with the results of NNLO

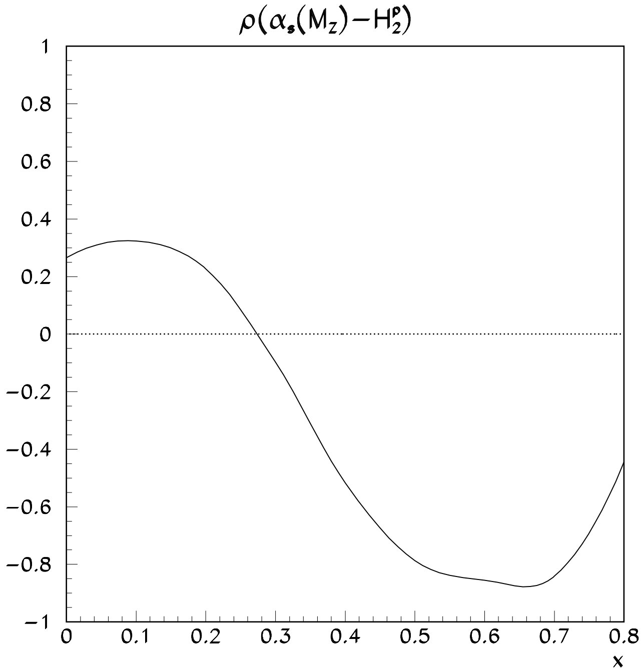

Figure 2: Shown is the correlation coefficient for the fitted values of $\alpha_s(M_Z)$ and H_2^p at different x.

3. Uncertainty due to the high-twist contribution

The HT terms can be taken into account using different approaches. Firstly, one can parameterize the HT terms by a flexible model independent function. In this approach possible uncertainties in variation of the HT terms allowed by data merge into the total experimental error in α_s. Secondly, one can cut data with low momentum transfer Q and/or hadronic mass W. Evidently in the second approach the HT error vanishes, however the statistical error can rise because the QCD evolution introduces the largest effect at small Q and therefore this region is most sensitive to the value of α_s. Thirdly, one can take into account the HT terms using available theoretical models. In this approach the HT uncertainty is subject of belief in reliability of the model.

In the analysis of ref. [7] we parameterize the twist-4 terms in additive form, $F_{2,\mathrm{L}} = F_{2,\mathrm{L}}^{\mathrm{LT,TMC}} + H_{2,\mathrm{L}}(x)/Q^2$, where $F_{2,\mathrm{L}}^{\mathrm{LT,TMC}}$ are the LT terms corrected for the target mass effects [10]. The functions $H_{2,\mathrm{L}}$ are defined in a piece-linear form with fitted coefficients that provides the model-independent account of the HT terms. One does need to include the HT terms in the analysis since they improve quality of the fit ($\chi^2/NDP = 2521/2274$ with and $\chi^2/NDP = 2851/2274$ without them). Possible logarithmic Q-dependence of $H(x)$ at $x \to 1$ could decrease the fitted α_s value [11], however data used in our analysis are not sensitive to such modifications of $H(x)$ because of the cut on x. The value of α_s is

analysis by Santiago and Yndurain, but agrees with the estimates of ref. [3] and results of the NNLO fit of ref. [9].

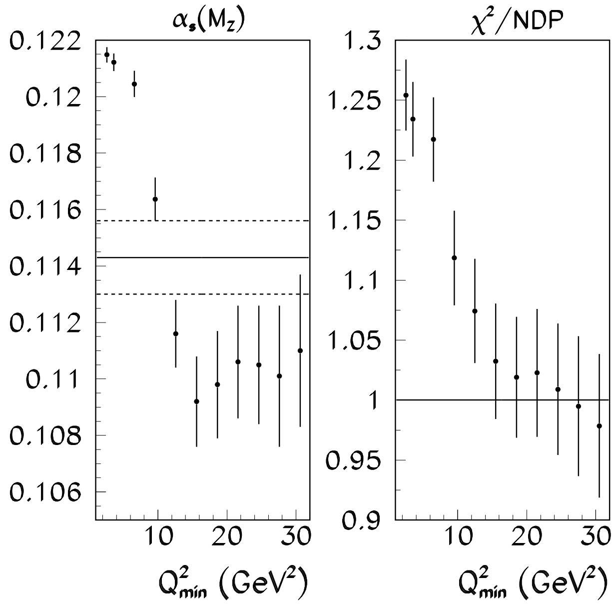

Figure 3: The values of $\alpha_s(M_Z)$ obtained from the fit with $H_{2,L} = 0$ and different cuts $Q_{\rm min}$ (left panel) and the corresponding values of χ^2/NDP (right panel). The horizontal lines in the left panel correspond to the central value and the error band of $\alpha_s(M_Z)$ in the analysis with the HT parameterized in the piece-linear form, the error bars in the right panel are $\sqrt{2/NDP}$.

strongly anti-correlated with H_2 at large x (see figure 2). For this reason the error in α_s is much larger than it could be in the absence of the HT terms. Indeed, in our trial fit with the HT terms set to 0, the error in $\alpha_s(M_Z)$ is 0.0003. Comparing this error with the error in α_s obtained in our analysis with the HT terms accounted for, we estimate the HT error in $\alpha_s(M_Z)$ to be 0.0014. This is the major source of the error in $\alpha_s^{\rm NNLO}$, more important than the estimated uncertainty due to the HO.

The results of scan fits with different cuts on Q and with no HT terms included are given in figure 3. These results suggest the optimal value of $Q_{\rm min} \sim 20$ GeV^2, which provide balance between stability of the central value of α_s and its error. For $Q^2_{\rm min} = 21.5$ GeV^2 the value of $\alpha_s(M_Z) = 0.1106 \pm 0.0020$ (exp.) was obtained. This value agrees with the results of analysis with the HT terms included that supports the both approaches. At that the value of α_s derived from the fit with the HT terms included is preferable since it is more robust due to the wider set of data was used in this case. The cut on W leaves out the data at small Q and large x and therefore makes fit less sensitive to the HT terms however not so efficiently as the cut on Q since the HT terms are important for moderate x as well. With the increase of the minimal value of W the error in α_s gets inappreciably large earlier than its central value is sustained.

In the analysis by Santiago and Yndurain the value of α_s was obtained from the fit with the HT terms set to 0. The uncertainty in α_s caused by this constraint was estimated as a difference between the value of α_s obtained from the fit with the HT terms set to

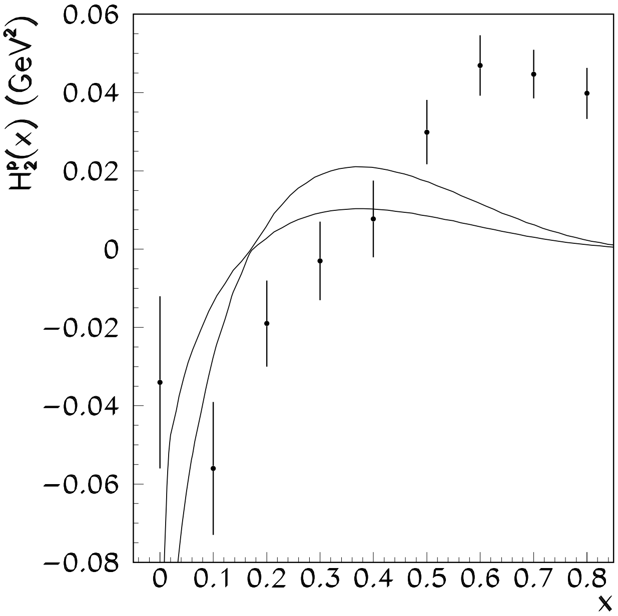

Figure 4: One-standard-deviation band for the HT term in the proton structure function F_2 obtained in the gradient model fit (curves) compared to the model independent determination (points).

0 and that of the fit with HT terms described by the gradient model. In this model the HT contribution to F_2 reads $A_2[dF_2^{NS}(x)/dx]/Q^2$, where F_2^{NS} is the non-singlet structure function and A_2 is a normalization parameter determined from data.

We find $A_2 = -0.018 \pm 0.006$ GeV^2 in the trail fit with $H_2(x)$ described by the gradient model[2]. We find that in this fit $\alpha_s(M_Z)$ changes by 0.0006 with respect to that of the fit with HT terms set to 0. This value agrees with 0.0004 obtained in the analysis by Santiago and Yndurain. However this shift in α_s cannot be considered as a reliable estimate of the HT uncertainty in α_s, since the gradient model describes the HT terms very poorly ($\chi^2/NDP = 2543/2067$). This is not surprising, because the gradient model can only be justified at $x \to 1$ and cannot be applied in the whole region of x (see figure 4). A correct estimate of the HT uncertainty in α_s in the analysis by Santiago and Yndurain would be given by a variation of $\alpha_s(M_Z)$ due to going from the fit with the HT terms set to 0 to the fit with the HT terms parameterized in the model-independent way. In our analysis such variation is -0.0069, much larger than estimate of the HT uncertainty based on the gradient model.

Taking into account this variation, we conclude that in fact eq. (1.1) disagrees with eq. (1.2) This disagreement is evidently connected with the substantial difference between the gluon distributions obtained in different fits. The gluon distribution was estimated in the analysis by Santiago and Yndurain as $\sim x^{-0.44}(1-x)^{8.1}$ [12] with the momentum

[2] In the trial fit we leave out the data points with $Q^2 < 3.5$ GeV^2 in order to better match the data set used in ref. [8].

carried by gluons 0.752 ± 0.014 at $Q^2 = 12$ GeV^2. This is much larger than the typical value of ~ 0.4 obtained in the recent global fits of PDFs [7, 13]. Independent estimation of this quantity based on the results of CDHS collaboration [14] is 0.56 ± 0.02, also much lower than the result by Santiago and Yndurain. We comment that due to the value of α_s is anti-correlated with the gluon distribution at small x, the increase in α_s caused by the absence of the HT terms in that analysis is compensated by its decrease due to enhanced gluons.

4. Conclusion

We also estimate theoretical errors connected with account of the heavy quarks contribution. These errors are more important than in our previous determination of α_s due to growing impact of the recent low-x HERA data. The uncertainty in $\alpha_s(M_Z)$ due to choice between the fixed-flavor-number and the variable-flavor-number factorization schemes is estimated as 0.0007. The variation of the c-quark mass by 0.25 GeV leads to the variation in $\alpha_s(M_Z)$ of 0.0006. Other possible sources of theoretical errors, including uncertainties in the NNLO evolution kernel and in the strange quarks distribution, are not considered because they give much smaller effect. Summarizing all sources of theoretical errors we get

$$\alpha_s^{\mathrm{NNLO}}(M_Z) = 0.1143 \pm 0.0014(\mathrm{exp.}) \pm 0.0013(\mathrm{theor.})\,.$$

This value is somewhat lower than the world average value of ref. [15] but agrees with it within errors.

Acknowledgments

I am indebted to S.A. Kulagin for careful reading of manuscript and valuable comments, D.V. Shirkov, and A. Vogt for stimulating discussions, and F.J. Yndurain for communication of the details of his analysis. I am grateful to the Theory Division of CERN, where the part of this work was done, for a hospitality.

References

[1] A. Retey and J.A.M. Vermaseren, *Some higher moments of deep inelastic structure functions at next-to-next-to leading order of perturbative QCD*, *Nucl. Phys.* **B 604** (2001) 281 [hep-ph/0007294].

[2] W.L. van Neerven and A. Vogt, *Improved approximations for the three-loop splitting functions in QCD*, *Phys. Lett.* **B 490** (2000) 111 [hep-ph/0007362].

[3] W.L. van Neerven and A. Vogt, *NNLO evolution of deep-inelastic structure functions: the non-singlet case*, *Nucl. Phys.* **B 568** (2000) 263 [hep-ph/9907472].

[4] K.G. Wilson, *Nonlagrangian models of current algebra*, *Phys. Rev.* **179** (1969) 1499

[5] L.W. Whitlow, E.M. Riordan, S. Dasu, S. Rock and A. Bodek, *Precise measurements of the proton and deuteron structure functions from a global analysis of the slac deep inelastic electron scattering cross-sections*, *Phys. Lett.* **B 282** (1992) 475;

BCDMS collaboration, A.C. Benvenuti et al., *A high statistics measurement of the proton structure functions $F_2(x, Q^2)$ and R from deep inelastic muon scattering at high Q^2*, *Phys. Lett.* **B 223** (1989) 485; *A high statistics measurement of the deuteron structure functions $F_2(x, Q^2)$ and R from deep inelastic muon scattering at high Q^2*, *Phys. Lett.* **B 237** (1990) 592;
New Muon collaboration, M. Arneodo et al., *Measurement of the proton and deuteron structure functions, F_2^p and F_2^d and of the ratio σ_L/σ_T*, *Nucl. Phys.* **B 483** (1997) 3 [hep-ph/9610231];
H1 collaboration, C. Adloff et al., *Deep-inelastic inclusive ep scattering at low x and a determination of α_s*, *Eur. Phys. J.* **C 21** (2001) 33 [hep-ex/0012053];
ZEUS collaboration, S. Chekanov et al., *Measurement of the neutral current cross section and f2 structure function for deep inelastic e^+p scattering at HERA*, *Eur. Phys. J.* **C 21** (2001) 443 [hep-ex/0105090].

[6] S.I. Alekhin, *Global fit to the charged leptons dis data: α_s, parton distributions and high twists*, *Phys. Rev.* **D 63** (2001) 094022 [hep-ph/0011002].

[7] S. Alekhin, *Parton distributions from deep-inelastic scattering data*, hep-ph/0211096.

[8] J. Santiago and F.J. Yndurain, *Improved calculation of F_2 in electroproduction and xF_3 in neutrino scattering to NNLO and determination of α_s*, *Nucl. Phys.* **B 611** (2001) 447 [hep-ph/0102247].

[9] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *NNLO global parton analysis*, *Phys. Lett.* **B 531** (2002) 216 [hep-ph/0201127].

[10] H. Georgi and H.D. Politzer, *Freedom at moderate energies: masses in color dynamics*, *Phys. Rev.* **D 14** (1976) 1829.

[11] E. Gardi and R.G. Roberts, *The interplay between sudakov resummation, renormalons and higher twist in deep inelastic scattering*, hep-ph/0210429.

[12] J. Santiago and F.J. Yndurain, *Nnlo calculation of dis: precision determination of the strong coupling constant, extraction of the gluon density and comments on 'hidden' gluinos*, *Nucl. Phys.* **86** *(Proc. Suppl.)* (2000) 69 [hep-ph/9907387].

[13] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *Uncertainties of predictions from parton distributions, I. Experimental errors*, hep-ph/0211080;
J. Pumplin et al., *New generation of parton distributions with uncertainties from global QCD analysis*, *J. High Energy Phys.* **07** (2002) 012 [hep-ph/0201195].

[14] J.G.H. de Groot et al., *Inclusive interactions of high-energy neutrinos and anti-neutrinos in iron*, *Z. Physik* **C 1** (1979) 143.

[15] S. Bethke, *Determination of the QCD coupling α_s*, *J. Phys.* **G 26** (2000) R27.